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Original article

It pays to base parameter estimation on a realistic description of model errors

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Abstract – The goal of this paper is to compare the profitability of nitrogen dose recommendations based on two models, which differ only in the statistical description of the model error and, consequently, in the way the parameter values are estimated. In the first model, the parameters were supposed fixed and their values were estimated by least squares. In the second model, the parameters were defined as random variables with a normal distribution. Optimal nitrogen rates were calculated for three types of gross margin and for five nitrogen prices by using the two fitted models. An evaluation of the profitability of the optimal nitrogen rates showed that the gross margins obtained by applying the nitrogen rates calculated with the random parameter model were higher by 53 to 319 F·ha^{-1} than the gross margins obtained by applying the nitrogen rates calculated with the fixed parameter model.

model evaluation / optimal nitrogen rate / parameter estimation / random parameter model

Résumé – Il est profitable d'estimer les paramètres en se basant sur une description réaliste des erreurs des modèles. Le but de cet article est de comparer les valeurs économiques de doses d'engrais optimales calculées à l'aide de deux modèles basés sur deux méthodes d'estimation différentes. Dans le premier modèle, les paramètres sont supposés fixes et sont estimés par les moindres carrés. Dans le deuxième modèle, les paramètres sont définis comme des variables aléatoires distribuées selon une loi Normale. Des doses optimales sont calculées en utilisant les deux modèles pour trois types de marge brute et pour cinq prix de l'engrais. Une évaluation de la qualité des doses optimales montre que les marges obtenues en appliquant les doses calculées avec le modèle à paramètres aléatoires sont supérieures de 53 à 319 F·ha^{-1} aux marges obtenues en appliquant les doses calculées avec le modèle à paramètres fixes.

dose d'engrais azoté optimale / estimation des paramètres / évaluation de modèles / modèle à paramètres aléatoires

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1. INTRODUCTION

Models of response to applied nitrogen (N) are often used for calculating optimal N doses [1, 6–8, 16, 19]. Numerous studies have been performed to define functions that describe properly the responses to applied N of crop characteristics, including yield [1, 6–8, 14, 16, 19], grain protein content [10, 14, 17], and residual mineral soil N at harvest [11, 14]. A related problem that has received relatively little attention is how to estimate the values of the model parameters. This problem must be solved before using the model to derive practical recommendations. Often, one assumes that the parameters take fixed values and that all the model errors are independent. This leads to estimating the parameter values by ordinary least squares [16, 19]. However, these assumptions are not realistic because, in N fertilizer trials, several measurements are usually made on the same site and year at different N doses. It is not reasonable to assume that the model errors are independent for the same site-year.

Several authors have proposed a different method for estimating parameters of models of response to applied N based on the use of a random parameter model [3, 5, 15, 22, 23]. In this kind of model, the form of the modeled response is common to all site-years, but the parameter values are assumed to vary between site-years. Typically, the model parameters are treated as random variables following a Normal distribution. The elements to be estimated are then the expected values and the variance-covariance matrix of the parameter distribution. These elements describe how the parameter values vary from site-year to site-year. Wallach [22, 23] argued that random parameter models provide a realistic yet simple way of taking into account the correlations between model errors in the same site-year. Our data do indeed seem consistent with a random parameter model (Fig. 1). For each type of measurement, the responses to applied N observed for the different site-years could be described by using the same functional form for all site-years but with different parameter values. Parameter variability can be due to various physical and biological factors that influence crop nitrogen uptake and crop nitrogen requirement like, for instance, soil and climate characteristics, or disease occurrence. Thus, it seems rather natural in principal to use random parameter models for modeling the response to applied N. However, we do not know if random parameter models are really superior in practice for

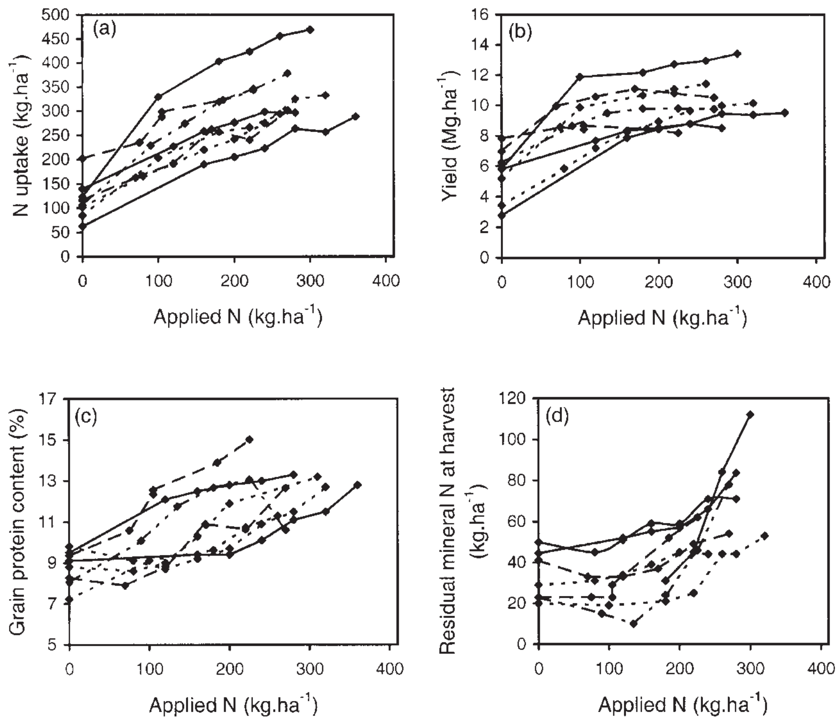

Figure 1. Measurements of nitrogen uptake (a), yield (b), grain protein content (c) and residual mineral soil nitrogen at harvest (d) as a function of applied nitrogen for eight site-years.

making N dose recommendations. In particular, we do not know if optimal N rates calculated by using a random parameter model are more profitable than those calculated by using a model with fixed parameters.

The goal of this paper is to evaluate the importance of using a random parameter model for calculating optimal N rates for winter wheat (*Triticum aestivum* L.). To do so, we compare two models for yield, grain protein content and residual mineral nitrogen, based on the same equations and based on the same data. The difference between the two models can be thought of in terms of alternative statistical descriptions of the errors. In one model, the errors for a given site-year are independent, but the parameter values vary between site-years, whereas the other assumes that all the errors are independent and that the parameters are fixed. These two models are used to calculate optimal N rates for various objective functions representing different types of farmers' gross margins. A non-parametric method [2] is then used to compare the calculated optimal N rates with data. The criterion for the comparison is the expected value of the objective function that would result from applying the optimal N rates.

2. MATERIALS AND METHODS

2.1. Data

The data used in this study were from 112 winter wheat experiments carried out between 1990 and 1999 on commercial farms in the Paris Basin, France. Each experiment was conducted at a different site, with recording made yearly, and consisted of five to eight different nitrogen treatments corresponding to different N fertilizer rates. The total number of N treatments in the data set is equal to 670 (112 site-years (5 to 8 N treatments). Two soil types were represented, a loam soil and a chalky soil. Common winter wheat varieties were used. N fertilizer was applied in two applications during the growing season. For each N treatment, yield (adjusted to 150 g·kg^{-1} grain moisture content) was measured at harvest. In addition, N uptake (amount of N in grain, straw, and root at harvest), grain protein content at harvest, and residual mineral N in the soil at harvest (NO_3^- plus NH_4^+) at 0- to 90-cm depth were measured for 484, 634, and 302 N treatments respectively. See Makowski et al. [14] for a more complete description of the plant sampling and nitrogen analysis. Measurements of N uptake, yield, grain protein content, and residual mineral N at harvest were in the ranges 62.4–490.1 kg·ha^{-1}, 1.52–15.37 Mg·ha^{-1}, 6.15–15.9%, and 10–211 kg·ha^{-1} respectively. In each experiment, end-of-winter mineral soil N (NO_3^- plus NH_4^+) in the 0- to 90-cm layer was measured during tillering (February), before the N application. The values of end-of-winter mineral soil N were in the range 15–180 kg·ha^{-1}. Figure 1 shows the values of yield, N uptake, grain protein content and residual mineral N at harvest, obtained in eight experiments for different N treatments.

2.2. Response functions

The functions proposed by Makowski et al. [14] are used to describe the responses to applied N of nitrogen uptake, yield, grain protein and residual mineral N at harvest (Tab. I). These functions were found to give satisfactory fit to the data [14]. The parameters Y_{MAX}, B, C_0, and U_0 represent physically meaningful quantities [21], namely maximal yield, N requirement per yield unit, maximal N recovery and N uptake without fertilizer application, respectively. Following Makowski et al. [15], we have introduced only a single field characteristic into the model, namely end-of-winter mineral soil N (noted ω) which is related to N uptake in the absence of applied N by

$$U_0 = \alpha_1 + \alpha_2\,\omega. \qquad (1)$$

We note θ the parameter vector: $\theta = [\alpha_1, \alpha_2, C_0, Y_{MAX}, A, B, T, P_1, P_2, R_{MIN}, R\,]^t$, where ^{t} denotes the transpose matrix operator.

2.3. Parameter estimation

Two statistical methods are used to estimate the parameters of the response functions presented in Table I. In the first method, θ is estimated by least squares [4]. This method assumes that θ is fixed and that all the model errors are independent. The variance of the error is assumed to be the same for all site-years and doses, but the four response variables may have different error variances. We note $\hat{\theta}$ the least squares estimate of θ. $\hat{\theta}$ and the four error variances are calculated from the data by using the S-PLUS function GNLS [18]. The elements of $\hat{\theta}$ are presented in Table II. The model based on $\hat{\theta}$ is noted MOD-FIXED. Note that, for MOD-FIXED, yield as a function of applied N has a plateau Y_{MAX} that begins at the dose $X_{MAX} = (BY_{MAX} - U_0) / C_0$. X_{MAX} depends on end-of-winter mineral soil N through (1). On the average over the end-of-winter mineral soil N ω of our data set,

Table I. Functions for nitrogen uptake (u), yield (y), grain protein content (p) and residual mineral nitrogen at harvest (r) versus applied nitrogen (x).

Variable	Function	
Nitrogen uptake	$u(x) = C_0 x + U_0$	if $x < X_{MAX}$,
	$u(x) = BY_{MAX} + \frac{C_0(x - X_{MAX})}{1 + T(x - X_{MAX})}$	if $x \geq X_{MAX}$
	with $X_{MAX} = (BY_{MAX} - U_0)/C_0$.	
Yield	$y(x) = Y_{MAX} + A[U(x) - BY_{MAX}] = Y_{MAX} + A[C_0 x + U_0 - BY_{MAX}]$	if $x < X_{MAX}$,
	$y(x) = Y_{MAX}$	if $x \geq X_{MAX}$.
Protein	$p(x) = P_1 + P_2\left(\frac{u(x)}{y(x)}\right)$	
	$= P_1 + P_2\left(\frac{C_0 x + U_0}{Y_{MAX} + A[C_0 x + U_0 - BY_{MAX}]}\right)$	if $x < X_{MAX}$,
	$= P_1 + P_2\left(\frac{BY_{MAX} \frac{C_0(x - X_{MAX})}{1 + T(x - X_{MAX})}}{Y_{MAX}}\right)$	if $x \geq X_{MAX}$.
Residual nitrogen at harvest	$r(x) = R_{MIN}$	if $x < X_{MAX}$,
	$r(x) = R_{MIN} + R(x - X_{MAX})$	if $x \geq X_{MAX}$.

Parameters: U_0 (nitrogen uptake without fertilizer application), C_0 (maximal nitrogen recovery), Y_{MAX} (maximal yield), A, B (nitrogen requirement per yield unit), T, P_1, P_2, R_{MIN} (minimal residual nitrogen at harvest), R.

Table II. Estimates of the parameters of model MOD-FIXED.

Parameter	Value
α_1	54.82 kg·ha^{-1}
α_2	0.97 kg·kg^{-1}
C_0	1.0 kg·kg^{-1}
Y_{MAX}	9.43 Mg·ha^{-1}
A	0.03 kg·kg^{-1}
B	25.8 kg·Mg^{-1}
T	0.0066 ha·kg^{-1}
P_1	3.53%
P_2	0.25 Mg·kg^{-1}
R_{MIN}	36.13 kg·ha^{-1}
R	0.23 kg·kg^{-1}

X_{MAX} = 120 kg·ha^{-1}. Table IV shows the values of the error variances estimated with GNLS.

In the second method, the response model is treated as a random parameter model [9, 18]. This model is noted MOD-RAND. Following Makowski et al. [15], we treat parameters T and α_2 as fixed and the nine other parameters as random. The nine random parameters are supposed to be a random vector with a Normal distribution N(μ, Σ), where μ is the (9×1) vector of expectations and Σ is the (9×9) variance-covariance matrix of the nine random parameters. The diagonal elements of Σ are the variances of the elements θ, indicating how widely they vary from site-year to site-year. The off-diagonal elements of Σ are the covariances between the elements of θ indicating the extent to which they tend to vary together. Makowski et al. [15] argue that only a few of the covariances between the random parameters need be

Table III. Estimates of the expected values, variances, and covariances of the parameters of model MOD-RAND. α_2 and T are assumed to be fixed.

Expectation		Variance		Covariance	
Name	Value	Name	Value	Name	Value
$E(\alpha_1)$	62.19 kg·ha^{-1}	$var(\alpha_1)$	845.5 kg^2·ha^{-2}	$cov(Y_{MAX}, B)$	–2.9
$E(C_0)$	0.99 kg·kg^{-1}	$var(C_0)$	0.033 kg^2·kg^{-2}	$cov(U_0, C_0)$	3.11
$E(Y_{MAX})$	9.58 Mg·ha^{-1}	$var(Y_{MAX})$	2.54 Mg^2·ha^{-2}	$cov(U_0, R_{MIN})$	–133.0
$E(A)$	0.03 kg·kg^{-1}	$var(A)$	0.000047 kg^2·kg^{-2}	$cov(U_0, R)$	–0.73
$E(B)$	25.9 kg·Mg^{-1}	$var(B)$	8.58 kg^2·Mg^{-2}	$cov(C_0, R_{MIN})$	–1.88
$E(P_1)$	3.58%	$var(P_1)$	3.97 (%^{2})	$cov(C_0, R)$	0.008
$E(P_2)$	0.25 Mg·kg^{-1}	$var(P_2)$	0.0042 Mg^2·kg^{-2}	$cov(R_{MIN}, R)$	–0.61
$E(R_{MIN})$	37.40 kg·ha^{-1}	$var(R_{MIN})$	290.9 kg^2·ha^{-2}		
$E(R)$	0.26 kg·kg^{-1}	$var(R)$	0.029 kg^2·kg^{-2}		
α_2	0.79 kg·kg^{-1}				
T	0.0039 ha·kg^{-1}				

Table IV. Estimates of the variances of the errors associated with the four response variables in the random parameter model (MOD-RAND) and in the fixed parameter model (MOD-FIXED).

Response variable	Error variance in MOD-FIXED	Error variance in MOD-RAND
N uptake	2329 kg^2·ha^{-2}	314 kg^2·ha^{-2}
Yield	2.45 Mg^2·ha^{-2}	0.18 Mg^2·ha^{-2}
Grain protein	1.8%^{2}	0.67%^{2}
Residual N at harvest	464.7 kg^2·ha^{-2}	85 kg^2·ha^{-2}

considered, the others can reasonably be taken as null. Eight covariances are thus estimated, namely cov(Y_{MAX}, B), cov(U_0, C_0), cov(U_0, R_{MIN}), cov(U_0, R), cov(C_0, R_{MIN}), cov(C_0, R), cov(R_{MIN}, R) and cov(P_1, P_2). The first seven covariances were assumed non-zero based on agronomic considerations: Y_{MAX} and B tend to be correlated because the climate has an effect on these two parameters (Boiffin et al., 1981). Soil properties and root development can influence both N uptake and residual mineral N at harvest and, consequently, can induce correlations between U_0, C_0, R_{MIN}, and R (Meynard et al., 1981; Wibawa, 1992). The correlation between P_1 and P_2 was found to be very high and so these two parameters were considered correlated. In this method, the residual error represents the error when site-year specific parameters are used in the model. Once again, there are four residual error variances corresponding to the four response variables. The values of μ, Σ, T, α_2 and of the four residual error variances are estimated from data by using the S-PLUS function Non-Linear Mixed Effect model (NLME) [13, 18, 20]. The estimated values are presented in Tables III and IV.

A comparison of Tables II and III show that the least squares parameter estimates and the expected values of the random parameters are similar but not identical. The residual error variances obtained for the two models, on the other hand, are very different, as is expected (Tab. IV). These variances are much higher in MOD-FIXED because they represent all the variability of the model error, whereas in MOD-RAND an important part of the error variability is accounted for by the variability in the random parameters. Alternatively one should note that MOD-RAND has 9×9 additional parameters which account for the variability between site-years.

2.4. Calculation of optimal nitrogen rates

MOD-FIXED and MOD-RAND are each used to calculate optimal rates for three different objective functions related to gross margin [15]. The first is defined by

$$m_1(x) = q\, y(x) - c\, x$$

where *q* is the grain price and *c* is the fertilizer cost. *q* is fixed to 700 F·Mg^{-1} (value corresponding approximately to the current grain price in France) and *c* is fixed successively to four values: 2, 2.5, 3, 3.5 and 4 F·kg^{-1}. Current fertilizer price is approximately 2–3 F·kg^{-1}. The higher prices are considered in order to simulate environmental taxes [12] or an increase in the price of oil. The second type of objective function is

$$m_2(x) = g[p(x)]\, y(x) - c\, x$$

where $g[p(x)]$ is a grain price function that depends on grain protein content, $p(x)$, as

$g[p(x)] = 750$ if $p(x) \geq 13\%$

$g[p(x)] = 700 + 20[p(x) - 10.5]$ if $13\% > p(x) \geq 10.5\%$

$g[p(x)] = 700$ if $p(x) \leq 10.5\%$.

The function $g[p(x)]$ is based on actual practice by a French agricultural cooperative, which increases the price paid to farmers for grain with a protein content above 10.5%. Here again, *c* is fixed successively at 2, 2.5, 3, 3.5 and 4 F·kg^{-1} in m_2. The third objective function is

$$m_3(x) = q\, y(x) - c\, x - h[r(x)]$$

where the grain price *q* is fixed at 700 F·Mg^{-1} and $h[r(x)]$ is a penalization function that depends on the residual mineral nitrogen, $r(x)$, as

$h[r(x)] = 0$ if $r(x) \leq 35$ kg·ha^{-1}

$h[r(x)] = 250$ if $r(x) > 35$ kg·ha^{-1}.

Currently no such penalty exists in France, but the cost of 250 F·ha^{-1} seems reasonable since that is approximately the cost of planting a cover crop during winter to avoid excessive leaching of nitrate. The threshold of 35 kg·ha^{-1} was chosen because that is about the level of residual mineral N that induces a water nitrate concentration higher than the permissible NO_3^- concentration of 50 mg·l^{-1} for drinking water in Europe. The fertilizer cost *c* is again fixed successively at 2, 2.5, 3, 3.5 and 4 F·kg^{-1}.

Each model is used to calculate an optimal rate for each objective function, for each fertilizer price, and for each site-year in the data set. Optimal N rates are calculated in function of measured values of end-of-winter mineral soil N. For example, the optimal rate X_{OPTi} for the gross margin $m_1(x)$ and for the site-year *i* is the value of *x* that maximizes $\hat{m}_{1i}(x) = q\hat{y}_i(x) - cx$, where $\hat{y}_i(x)$ is the yield value predicted by the model for the *i*th site-year in the data set in function of ω_i. With MOD-FIXED, $\hat{y}_i(x)$ is calculated simply by replacing the function parameters by their estimated values:

$$\hat{y}_i(x) = \hat{Y}_{MAX} + \hat{A}[\hat{C}_0 x + \hat{U}_{0i} - \hat{B}\hat{Y}_{MAX}] \quad \text{if } x < (\hat{B}\hat{Y}_{MAX} - \hat{U}_{0i})/\hat{C}_0$$

$$\hat{y}_i(x) = \hat{Y}_{MAX} \quad \text{if } x \geq (\hat{B}\hat{Y}_{MAX} - \hat{U}_{0i})/\hat{C}_0$$

where $\hat{U}_{0i} = \hat{\alpha}_1 + \hat{\alpha}_2\omega_i$ (the notation ^ means 'estimated value') and ω_i is the end-of-winter mineral soil N measured at site-year *i*. As $\hat{U}_{0i}$ depends on ω_i, X_{OPTi} depends on ω_i as well. With MOD-RAND, $\hat{y}_i(x)$ is calculated by generating 200 parameter values from N($\hat{\mu}$, $\hat{\Sigma}$) and by taking the average yield:

$$\hat{y}_i(x) = \frac{1}{200}\sum_{j=1}^{200} min\left\{Y_{MAXj} + A_j[C_{0j}x + U_{0ij} - B_j Y_{MAXj}]; Y_{MAXj}\right\} \quad (2)$$

where $U_{0ij} = \alpha_{1j} + \hat{\alpha}_2\omega_i$, and $Y_{MAXj}, A_j, C_{0j}, B_j, \alpha_{1j}$ are the *j*th values generated from N($\hat{\mu}$, $\hat{\Sigma}$). Thus, with MOD-RAND, $\hat{y}_i(x)$ represents an average yield value calculated over parameter values for a given ω_i. This approach allows us to calculate optimal N rates in function of measured values of end-of-winter mineral soil N. The same approach is used to calculate optimal N rates for gross margins $m_2(x)$ and $m_3(x)$.

2.5. Estimation of the gross margin values obtained by applying the calculated optimal nitrogen rates

The method of Antoniadou and Wallach [2] is used to estimate the expected gross margin values that would be obtained if the calculated optimal N rates were applied. Expected gross margin values are estimated directly from data using local quadratic regressions. Very roughly, the method uses a non-parametric interpolation between the doses actually observed, in order to estimate what income would result from applying the calculated optimal N rates. This approach is not based on assuming that a particular parametric model is correct and so does not favor any of the two models. A local regression is performed for each model, for each gross margin type and for each N price by using the LOESS function of S-PLUS with a span fixed to 1 (i.e. 100% of the measurements were used to perform each regression) and with a tricube weight function [20]. F tests showed that a smaller span value is not justified. Information on the accuracy of the local regressions are provided by the

LOESS routine through the standard deviations of the estimated gross margin values.

3. RESULTS-DISCUSSION

3.1. Optimal nitrogen rates

Figure 2 shows the average optimal N rates calculated with MOD-FIXED and MOD-RAND. Each average value was obtained by averaging 112 optimal N rates calculated in function of end-of-winter mineral soil N for the different site-years of the data set. For MOD-FIXED, the average optimal N rate is 120 kg·ha^{-1} for all three objective functions and all fertilizer prices, with the single exception of objective function m_2 and fertilizer price at 2 F·kg^{-1}. The dose 120 kg·ha^{-1} is the average value of the dose that maximizes yield over the 112 site-years of the data set. The reason that the optimal dose is almost always the dose maximizing yield is the form of the response functions. Below X_{MAX}, where the yield plateau begins, increasing N leads to a relatively large increase in yield and no increase in residual mineral soil N at harvest and so is always worthwhile for objective functions m_1 and m_3. Above X_{MAX}, increasing N leads to no increase in yield. Only in the case of m_2 can it be worthwhile to exceed X_{MAX}, since this can lead to an increase in protein content.

The optimal N rates calculated using MOD-RAND vary depending on the objective function and the fertilizer price (Fig. 2). The average optimal N rates calculated for gross margin m_1 with MOD-RAND are in the range 179–216 kg·ha^{-1}, depending on the N price (Fig. 2a). The average optimal N rate values obtained for gross margin m_2 are higher (203–248 kg·ha^{-1}) (Fig. 2b) while the values obtained for gross margin m_3 are lower (173–211 kg·ha^{-1}) (Fig. 2c). For the three gross margin types, the optimal rates calculated with MOD-RAND decrease when the N fertilizer price increases. The average optimal N rates calculated with MOD-RAND are higher than those calculated with MOD-FIXED by 35 to 112 kg·ha^{-1} depending on the objective function and on the fertilizer price. For gross margins m_1 and m_3, the difference between the two models tends to decrease when the N price increases. These results are consistent with

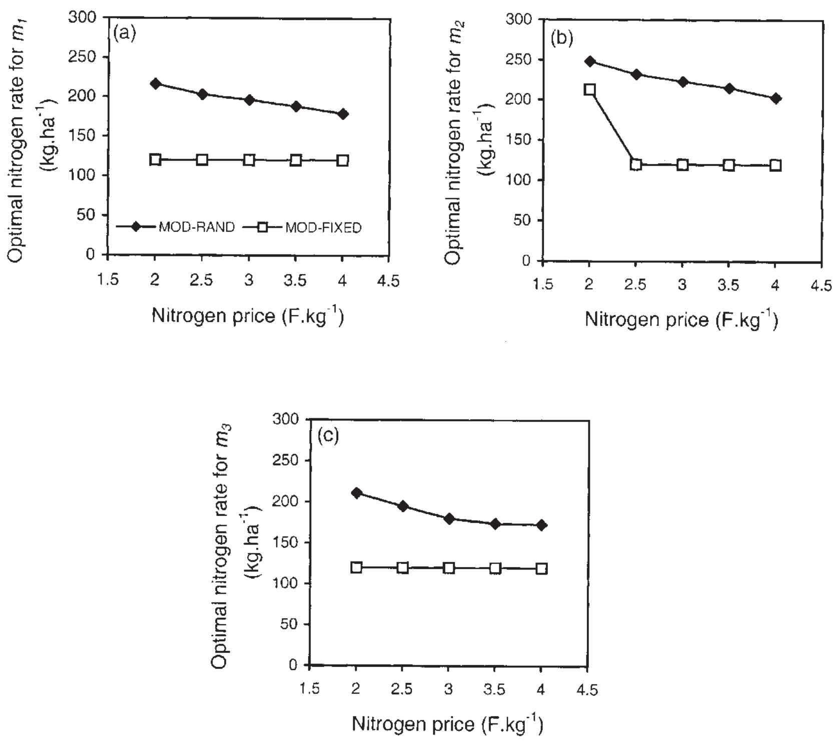

Figure 2. Average optimal nitrogen rates calculated using MOD-RAND and MOD-FIXED for gross margins m_1 (a), m_2 (b), m_3 (c) and for five different fertilizer prices. Optimal rates were averaged over the 112 site-years of the data set.

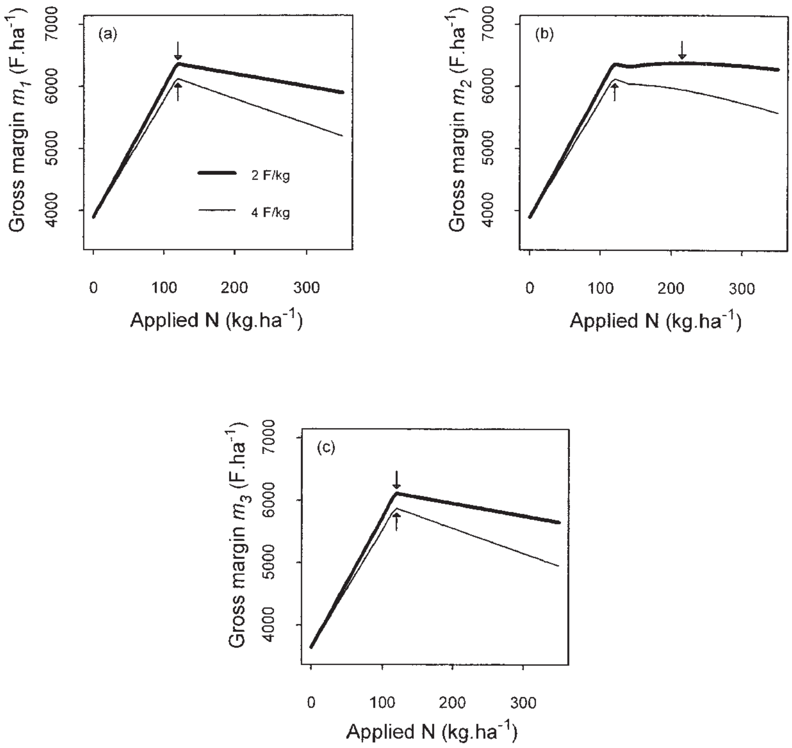

Figure 3. Responses of gross margins m_1 (a), m_2 (b) and m_3 (c) to applied nitrogen predicted by using MOD-FIXED for a site-year with 65 kg·ha^{-1} of end-of-winter mineral soil nitrogen and for two fertilizer prices. Arrows indicate maximal gross margin values.

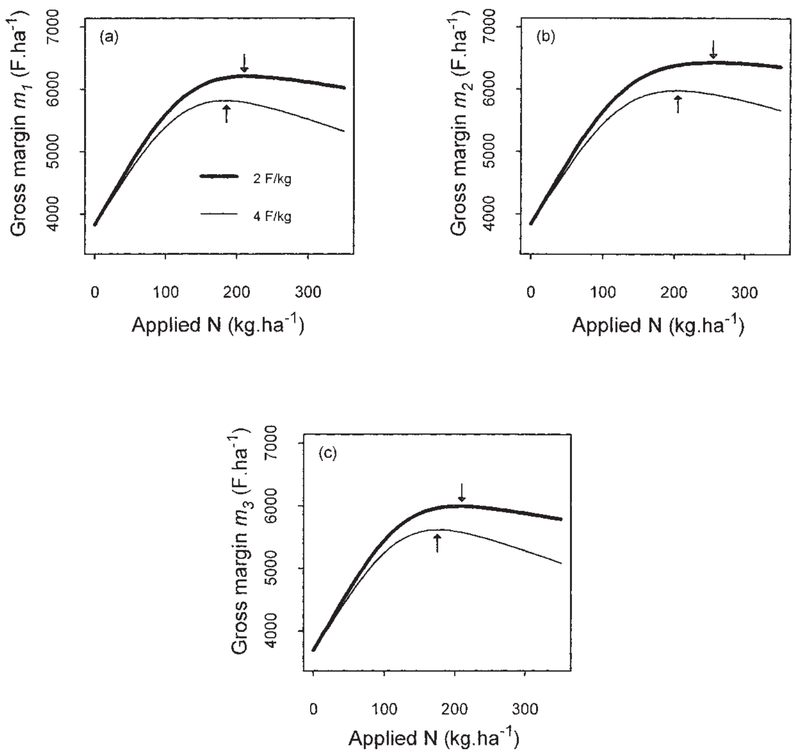

Figure 4. Responses of gross margins m_1 (a), m_2 (b) and m_3 (c) to applied nitrogen predicted by using MOD-RAND for a site-year with 65 kg·ha^{-1} of end-of-winter mineral soil nitrogen and for two fertilizer prices. Arrows indicate maximal gross margin values.

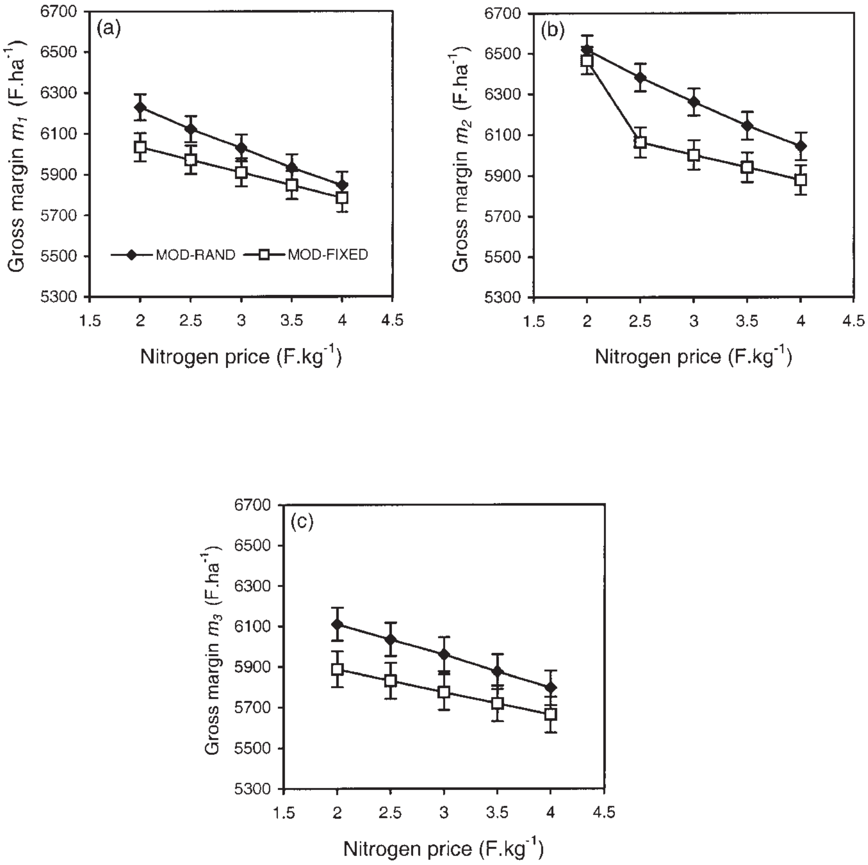

Figure 5. Non-parametric estimates (based on measurements) of the expected gross margin values that result from applying the optimal nitrogen rates calculated with MOD-RAND and MOD-FIXED. The error bars indicate standard deviations of the non-parametric estimates.

the results of Babcock [3]. That author showed analytically for a simple linear-plus-plateau yield response function that a random plateau (i.e. random maximal yield) induces an increase in the calculated optimal N rates if the price of N fertilizer is low relative to the price of the crop.

These large differences between MOD-RAND and MOD-FIXED are due to the consequence of assuming random parameters on the shape of the predicted gross margin response curves, as illustrated in Figures 3 and 4 for a site-year with ω = 65 kg·ha^{-1}. Figure 3 shows that the gross margin response curves predicted by MOD-FIXED have sharp breaks. For instance, the value of gross margin m_1 predicted by MOD-FIXED first increases linearly as a function of applied N and then decreases sharply when the N dose exceeds 125 kg·ha^{-1} (Fig. 3a). Because of the initial linear section of the gross margin response curve, the N rate maximizing gross margin m_1 is insensitive to the N price (Fig. 3a) and is always equal to the N rate maximizing yield $(\hat{B}\hat{Y}_{MAX} - \hat{U}_{0i}) / \hat{C}_0$. This rate is equal to 125 kg·ha^{-1} for the site-year considered in Figure 3 and is equal to 120 kg·ha^{-1} in average over the 112 site-years of the data set. On the contrary, the gross margin response curves predicted by MOD-RAND are all completely smooth (Fig. 4). This is due to the fact that, with MOD-RAND, each gross margin response curve is obtained by averaging 200 linear-plus-plateau response curves (Eq. (2)) resulting in a smooth average curve. Similar results were obtained analytically by Beck and Helfand [5] for a simple response function.

3.2. Objective function values

Figure 5 shows the non-parametric estimates of the expected gross margin values that would be obtained if the calculated optimal N rates were applied. Error bars represent one standard deviation, as calculated by the non-parametric regression program. The results discussed in the previous section show important differences between the optimal N rates calculated by using MOD-RAND and those calculated by using MOD-FIXED. We compare here the monetary results induced by the application of the optimal N rates calculated by the two models.

Figure 5 shows that the expected gross margin values obtained with MOD-RAND are higher than the values obtained with MOD-FIXED by 62 to 194 $F \cdot ha^{-1}$ for gross margin m_1, by 53 to 319 $F \cdot ha^{-1}$ for gross margin m_2, and by 133 to 222 $F \cdot ha^{-1}$ for gross margin m_3. For gross margins m_1 and m_3, the differences between the monetary results of the two models are particularly important when the N prices are below 3 $F \cdot kg^{-1}$ (Figs. 5a and 5c). For these N prices, the differences are larger than two standard deviations and so can be considered as significant. The difference between the monetary results obtained with the two models for gross margins m_1 and m_3 decreases when the N price increases. This is because the differences between the optimal N rates calculated for gross margins m_1 and m_3 with the two models are smaller for high N prices than for low N prices (Figs. 2a and 2c). For gross margin m_2, the differences between the monetary results obtained with MOD-RAND and with MOD-FIXED are larger than two standard deviations when the N price is above 2 $F \cdot kg^{-1}$. On the other hand, the values of m_2 obtained with MOD-RAND and MOD-FIXED are very similar when the N price is equal to 2 $F \cdot kg^{-1}$. This is because, as shown in Figure 2b, the optimal N rates obtained with the two models for this N price are not very different.

4. CONCLUSIONS

The results presented in this paper show that the statistical description of model error has a strong effect on the calculated N rates, and that the different calculated N rates have different profitability. The optimal N rates calculated with our response functions are higher by 35 to 112 $kg \cdot ha^{-1}$ if the model parameters are considered as random rather than fixed. This result reveals that two models based on the same functions and the same data but based on two different statistical descriptions of error can give very different N fertilizer recommendations. Another important result shown in this paper is that the N rates calculated with the random parameter model are more profitable by 53 to 319 $F \cdot ha^{-1}$ than the N rates obtained with the fixed parameter model. Thus, it is more interesting to use a random parameter model than a fixed parameter model for calculating optimal N rates. More generally, the results of this study indicate that the use of an appropriate statistical description of error is important for the quality of model-based decision rules.

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